Characterization of Transitional, High-Enthalpy Boundary Layers on a Slightly-Blunted Cone. Part I: Schlieren Imaging

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This paper presents an experimental investigation into the boundary-layer stability of a slender cone in hypervelocity flow within the T5 reflected-shock tunnel. Schlieren imaging was utilized to characterize the frequency content and disturbance structures experienced within the boundary layer of a 5° cone in a Mach-5 freestream with high reservoir enthalpy, 8-10 MJ/kg. The effects of varying freestream Reynolds number and nose bluntness were examined. Second-mode frequency peaks between 1200-1300 kHz were identified in all cases, but they persisted over a longer extent in the sharper-nose cases. The bluntest nosetip case exhibited unique nonmodal structures which extended into the freestream, and the signature of the second-mode appeared limited to the near-wall region. N factors contours suggested interplay between second-mode content and frequency content outside the second-mode band. Cross-bicoherence calculations indicated that, for the sharper nosetip, nonlinear interactions between the second-mode fundamental and its first harmonic persisted to the point of breakdown. For the blunter nosetip, dominant nonlinear interactions involved low-frequency content.

I. Introduction

The prediction of boundary-layer transition has repeatedly been placed in the forefront of design concerns when it comes to hypersonic vehicles. Identified as contributing the "largest uncertainty" to hypersonic plane design, the inability to predict this phenomenon can more than double the takeoff weight of an over-designed aircraft [1, 2]. While it is well-understood that the boundary layer is highly sensitive to thermal state of the freestream, many ground-test facilities fail to replicate the thermal conditions that vehicles experience in true atmospheric flight. Malik [3] demonstrates how the post-shock conditions around a reentry cone can induce temperatures five times the wall temperature. Stanfield et al. [4] suggested that this discrepancy in temperature ratio between flight and ground facilities can significantly change the transition location, even outweighing opposing effects from freestream noise. Thus, the effect that the temperature gradient within the boundary layer has on boundary-layer stability needs to be characterized.

A number of campaigns in high-enthalpy facilities have made observations related to the spectral content and linear growth of boundary-layer disturbances. Parziale et al. [5] used focused laser differential interferometry (FLDI) to measure the spectral content associated with the second-mode instability for low and high-enthalpy conditions. For air and CO_2 cases, measured frequencies were notably lower than those predicted by PSE calculations. Laurence et al. [6] visualized second-mode activity in low and high-enthalpy conditions using schlieren imaging, but full transition to turbulence was not captured. Marineau et al. [7] compared the spectral amplitudes and growth rates of second-mode disturbances in a variety of facilities, showing that, only at high enthalpies, the growth rate of these disturbances

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appeared to be sensitive to freestream Reynolds number. Thus, the nature of transition at high enthalpy needs to be further explored.

Along with knowledge gaps related to linear disturbance growth, even less is known about the nonlinear processes which accompany the transition to turbulence under high-enthalpy conditions. From a design perspective, nonlinear interactions are important because they determine the topology of heating experienced by a vehicle undergoing transition, such as the hot streaks identified on straight and flared cones in Refs. [8–12]. Numerical investigations have indicated that high-enthalpy nonlinear processes are an entanglement of effects related to phase-speed synchronization, three-dimensional modulation, and thermo-chemical non equilibrium [8, 13–15]. Scarce experimental analysis has been performed. Jewell et al. [16] measured the propagation and generation of turbulent spots at high enthalpy ($6 \le h_0 \le 12$ MJ/kg) and low wall-to-edge temperature ratios ($0.13 \le T_w/T_e \le 0.24$), but the mechanisms of breakdown were not explored. Chokani [17] compared the nonlinear second-mode development along a flared cone for an adiabatic and cooled-wall case using hot-wire measurements. Wall-cooling pushed nonlinear interactions upstream, but these experiments were limited to a reservoir temperature of 450 K. Ide et al. [14] assessed nonlinear interactions between low and high (second-mode) frequencies upstream of transition using PCB measurements, but they limited their campaign to a low-enthalpy (h = 3 MJ/kg) condition.

This study seeks to uncover the fluid mechanics present during the transition to turbulence on a slender cone at high enthalpy. High-speed schlieren imaging provides a full-field view of the breakdown process. Building on the results presented in Ref. [18], the frame rates available in this campaign allow for a frequency analysis based on time reconstructions of pixel signals within the boundary layer. Structures within the boundary layer and freestream are characterized using optical analysis, power-spectral-density estimation, and calculation of higher-order spectral content.

II. Facility & Setup

All testing was conducted in the T5 reflected-shock tunnel at the California Institute of Technology. The facility design and operation are detailed in Ref. [19] and summarized here briefly. Moving in the downstream direction, the tunnel can be segmented into the following components: secondary reservoir, piston, compression tube (CT), primary diaphragm, shock tube (ST), secondary diaphragm, contoured nozzle, test section, and dump tank. Before a shot, the test section, dump tank, and both tubes are evacuated. Then the ST is filled with air to desired test-gas pressure P_1 , and the CT is filled with an argon/helium mixture to the desired driver pressure P_{CT} . Finally the secondary reservoir, upstream of the piston, is filled with compressed air to a specified gage pressure P_{2R} , typically around 1200 psi. Once exposed to the pressure in the secondary reservoir, the 120-kg piston travels downstream, adiabatically compressing the driver gas mixture to a desired value P_4 . At this point the pressure difference between the driver gas in the CT and the test gas in the ST is high enough to burst the primary stainless-steel diaphram. A shock wave travels through the ST at a speed U_s , compressing the test gas, until it is reflected at the downstream end of the ST, bursting the secondary mylar diaphragm. Under tailored operation, the test gas is considered stagnant after being additionally compressed and heated from the shock reflection to an ultimate reservoir pressure P_R and temperature T_R . This flow is then accelerated through the axisymmetric nozzle to a Mach 5, and hypervelocity flow is established for about 1 ms in the test section. Total enthalpies h_0 between 8 – 10 MJ/kg were established for all shots, resulting in freestream-to-wall temperature ratios T_{∞}/T_{w} ranging from 4-5.

Table 1 lists the relevant reservoir and freestream conditions for the tests analyzed below. The total enthalpy and reservoir conditions were calculated using Cantera and the Shock & Detonation Toolbox [20].

The model was a 5°-half-angle cone, 99 cm in total length. The frustum section was aluminum and 83 cm in length, and the interchangeable nosetip was machined out of molybdenum. In this campaign, the nosetip radius, R_N , was also increased for the last shot, 2993, in an effort to observe any effects on the content within the boundary layer. For the majority of the tests, a 16-cm nosetip with nose radius $R_N = 2$ mm was used. For Shot 2993, a blunter $R_N = 3$ mm nosetip, 15 cm in length, was tested. The model was installed at zero incidence for all the shots, and the angle of attack was checked with a laser level mounted on the upstream side of the nozzle.

| | Reservoir | | | Freestream | | | | | | Nose Radius |
|------|-----------|---------|-------|--------------|--------------|--------------|----------------------------------|--------------|------------------|-------------|
| Shot | P_0 | h_0 | T_0 | U_{∞} | P_{∞} | T_{∞} | Re_{∞} | M_{∞} | T_{∞}/T_w | R_N |
| | (MPa) | (MJ/kg) | (K) | (m/s) | (kPa) | (K) | $(\times 10^{6} \text{ m}^{-1})$ | _ | - | (mm) |
| 2983 | 53.5 | 9.48 | 5963 | 3913 | 32.1 | 1472 | 5.33 | 5.05 | 5.0 | 2 |
| 2984 | 55.9 | 9.60 | 6021 | 3935 | 33.7 | 1495 | 5.48 | 5.04 | 5.1 | 2 |
| 2985 | 58.1 | 9.03 | 5794 | 3839 | 33.8 | 1388 | 6.07 | 5.11 | 4.7 | 2 |
| 2987 | 58.9 | 8.85 | 5724 | 3808 | 33.8 | 1354 | 6.28 | 5.13 | 4.6 | 2 |
| 2988 | 60.1 | 8.93 | 5759 | 3822 | 34.6 | 1369 | 6.35 | 5.12 | 4.6 | 2 |
| 2989 | 60.1 | 9.74 | 6098 | 3963 | 36.6 | 1523 | 5.80 | 5.02 | 5.2 | 2 |
| 2993 | 60.7 | 8.07 | 5396 | 3664 | 31.7 | 1191 | 7.04 | 5.28 | 4.0 | 3 |

Table 1 Test Conditions

The schlieren setup explained in Ref. [18] was employed here. Illumination was generated by a Cavilux HF laser, and an adjustable iris diaphragm was used to limit the amount of light in an effort to avoid saturation. The beam was expanded through a plano-convex lens, collimated by a parabolic mirror, and directed by a few planar mirrors through the test section. A parabolic mirror focused the beam back down to a point, where the knife edge was inserted. Finally, the beam passed through a bandpass filter, which prevented the test-gas luminosity from obscuring the signal, and then a series of long-focal-length plano-convex lenses, which were used to modify the magnification of the images. Fig. 1 depicts the resulting field of view (FOV) along the cone imaged with schlieren. Although the field of view did change slightly between shots, the images generally captured a 17×1 -cm region whose upstream edge was stationed 58 cm downstream of the nosetip. Experiments using FLDI, discussed in Ref. [21], also made measurements within this region. As shown in Fig. 1, the FLDI beams were focused at x = 680 mm along the cone surface.



Fig. 1 Location of diagnostics on cone

Table 2 lists the optical settings used for schlieren. In this campaign, two different cameras were used: a Phantom v2512 was used for the first three shots and then swapped out for a Phantom TMX 7510 for the remainder of the matrix. Both cameras provided higher frame rates than those available using the Phantom v2640 in Ref. [18], and the optical setup of each run could be modified to focus on features of interest. For example, the frame rate could be set to the maximum 875 kHz to maximize the number of images captured, or it could be lowered slightly to allow for increased laser pulse width and, thus, more light and a better signal-to-noise ratio.

| | | Frame | Exposure | Laser | Image | Spatial |
|------|---|-------|----------|-------------|------------------|------------|
| Shot | Camera | Rate | Time | Pulse Width | Resolution | Scale |
| | | (kHz) | (µs) | (ns) | $(px \times px)$ | (pixel/mm) |
| 2983 | | 362 | 2.3 | 30 | 1280×48 | 6.7 |
| 2984 | v2512 | 393 | 2.0 | 30 | 1152×48 | 6.7 |
| 2985 | | 393 | 2.0 | 30 | 1280×48 | 6.7 |
| 2987 | | 875 | 0.86 | 20 | 1280×64 | 7.1 |
| 2988 | TMV 7510 | 669 | 1.2 | 30 | 1280×64 | 6.7 |
| 2989 | 1 | 669 | 1.2 | 30 | 1280×64 | 9.6 |
| 2993 | | 669 | 1.2 | 30 | 1280×64 | 6.3 |

Table 2 Imaging Settings

III. Results

A. Boundary-layer structures

Examples of reference-subtracted schlieren sequences are shown in Fig. 2. Each shows the development of a unique feature. In Fig. 2(a), Shot 2983, at $Re_{\infty} = 5.33 \times 10^6$ m⁻¹, exhibits the propagation of second-mode wavepackets, similar to those seen in Ref. [18]. Two distinct appearances are visible: dark streaks that appear to lean downstream moving from x = 645 to 670 mm, and the more conventional rope-like packet propagating from x = 675 to 705 mm. In Fig. 2(b), Shot 2988 exemplifies the passage of a turbulent spot at $Re_{\infty} = 6.35 \times 10^6$ m⁻¹. Radiating streaks extending out of the boundary layer are visible, but the signature of second-mode waves appears to lead and follow the structure. This observation could be explained by the visualizations of Casper et al. [22], who found the signature of second-mode waves to persist even after the point of breakdown, encompassing arrowhead-shaped turbulent spots. In Fig. 2(c), Shot 2993, with an increased $R_N = 3$ mm, and at $Re_{\infty} = 7.04 \times 10^6$ m⁻¹, exhibits the presence of a nonmodal feature, such as that discussed in Ref. [23], moving from x = 670 mm to 695 mm. Optical filtering techniques provided the basis for calculation of boundary-layer height and disturbance propagation speeds, as discussed in Ref. [18]. Average boundary-layer heights ranged from $\delta = 1.0 - 1.3$ mm, with the average measured across the full frame from the sharper-nose shots being $\delta = 1.1$ mm. The average propagation speed was 3450 m/s, or 93% of the edge velocity. The measured δ from the blunter-nose case was also 1.1 mm, and the propagation speed was 3400 m/s, or 96% of the edge velocity.

B. Power Spectral Density

The methodology described in Refs. [24, 25] was employed here to analyze spectral content, especially that associated with second-mode instabilities. Temporally-resolved pixel signals were reconstructed using neighboring pixel information and the determined propagation speeds. The resulting pixel time signals were 23,000-35,000 points in length. Welch's method was used to generate power-spectral density (PSD) estimations for the reconstructed pixel signals. For these calculations, a Blackman window was applied to segments spanning 20,000 points.

Fig. 3 shows the resulting PSD estimates at various wall-normal heights, averaged over an approximately 20-mm region centered at x = 680 mm. The results shown, for Shots 2989, 2987, 2988, and 2993, are arranged in order of increasing freestream Reynolds number. As shown in Fig. 3(a), only a faint presence of the second-mode exists in the peaks near 1300 kHz for $0.2 \le y \le 0.6$ mm. In Fig. 3(b), Shot 2987 at a higher freestream Re_{∞} shows significant elevation of the spectral power at 1200 kHz for $0.2 \le y \le 0.5$ mm but the peaks fades for $y > \delta$. For Shot 2988 in Fig. 3(c), the strongest spectral peak sits right above the wall at f = 1250 kHz. Moving away from the wall, this peak persists but gradually skews toward lower frequencies. This analysis agrees very well with that performed at x = 680 mm using FLDI for a shot with the same Re_{∞} [21]. The second-mode peaks identified with schlieren and FLDI agree within 2% of one another. Finally, as shown in Fig. 3(d), the wall-normal spectra for Shot 2993 are distinct from the sharper-nose cases. A peak near 1200 kHz does sit at y = 0.2 mm but appears at no other wall-normal height. Moving farther from the wall, a peak near 950 kHz emerges, most notably at y = 1.6 mm. As suggested by Fig. 2(c), the introduction of nonmodal features distinguishes this shot from the $R_N = 2$ mm cases, whose spectra are clearly dominated by second-mode presence.



Fig. 2 Schlieren series from Shots 2983, 2988, and 2993



Fig. 3 Wall-normal frequency content from Shots 2984, 2987, 2988, and 2993

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N-factor contours were generated to provide insight into the spectral development throughout the field of view. The N factor N was defined as

$$N(x) = 1/2\ln p(x) + C,$$
(1)

where p is the PSD power of a particular frequency at a location x along the cone.

Fig. 4 shows resulting N factor contours for the frequency domain $800 \le f \le 3000$ kHz for five shots in order of increasing Re_{∞} . Figs. 4(a)-4(d) correspond to four shots with the sharper nosetip, $R_N = 2$ mm, whereas Fig. 4(e) shows the result from the blunter nosetip, $R_N = 3$ mm. Pixel signals right above the wall were used to generate the contours since, in general, these signals had the strongest signal-to-noise ratio and resulting spectral power. Although band-stop filters were applied to the reconstructed pixel signals, artifacts of the frame rate and its harmonics are still visible at certain points (i.e. f = 2000 and 2700 kHz at $Re = 4 \times 10^6$ in Shot 2989 and at $Re = 4.6 \times 10^6$ and 4.9×10^6 in Shot 2993) and should not be considered in the spectral analysis. As shown, all shots demonstrate frequency peaks corresponding to second-mode content near f = 1200 - 1300 kHz. In Figs. 4(a)-4(c) the second-mode peak can be seen slowly decreasing in frequency, and in Figs. 4(b) and 4(c), the frequency lobe amplifies in power noticeabley in the downstream half of the field of view. In Figs. 4(d) and 4(e), Shots 2988 and 2993 exhibit the breakdown to turbulence. In Fig. 4(d) the second-mode lobe expands and dissipates into a broad range of frequency content by $Re = 4.5 \times 10^6$. For the bluntest-nosetip case in Fig. 4(e), there is no elongated lobe of second-mode content; instead, the content which just develops near 1200 kHz at $Re = 4.6 \times 10^6$ quickly breaks down by $Re = 5 \times 10^6$. Peak frequencies were identified over Re ranges where second-mode content could be clearly identified. These peak frequencies were then fitted with a curve $f_{peak} = A/\sqrt{Re}$, where A is a constant. This fit was based on expected boundary-layer tuning and $\delta \propto \sqrt{x}$. For cases where the frequency did not decrease significantly within the field of view, the fit was relaxed to $f_{peak} = A/\sqrt{Re} + C$, where C was an additional constant. The resulting f_{peak} fits are displayed on the contours as dashed lines.

Despite the presence of frame-rate artifacts at various locations, the N factor contours also indicate the presence of higher- and lower-frequency content before and after the breakdown to turbulence. For example, near $Re = 4.15 \times 10^6$ in Shot 2989, lower-frequency content in the range 800-1200 kHz becomes significant, and some additional lobes around 1900 kHz emerge. Shot 2988 shows significant activity within $1500 \le f \le 2000$ kHz for most of the field of view. These contours motivate an investigation into whether such outer-band content modulates the second-mode development. Interactions between various frequencies are investigated in the following section.

C. Cross-Bicoherence

As described by Chokani in Ref. [17], nonlinear disturbances persist and evolve over long length scales at high Mach numbers. Bicoherence calculations have been identified as the "most direct method" of identifying nonlinear processes present within a signal [26]. This higher-order spectral analysis is used to assess the degree of phase locking between a triad of frequencies $\langle f_1, f_2, f_3 = f_1 + f_2 \rangle$. As a result, levels of high bicoherence indicate energy exchange between various modes present within a signal, and the analysis is instrumental in characterizing the nonlinear development of instabilities. Specifically, calculation of the cross-bicoherence between two spatially-separated signals, as in Ref. [14], allows the identification of sum and difference interactions within a finite region. This technique was explored for the schlieren data.

Similar to the power spectral density, which is the Fourier transform of the second-order moment, or autocorrelation function R_{qq} , the bispectrum is the Fourier transform of the third-order moment R_{qqq} . This moment is defined as

$$R_{qqq}(\tau_1, \tau_2) = E[q(t)q(t+\tau_1)q(t+\tau_2)],$$
(2)

where E[-] is the expectation operator and τ_1 and τ_2 are time delays. In Fourier space, the bispectrum is then defined as

$$B(f_1, f_2) = E[Q(f_1)Q(f_2)Q^*(f_1 + f_2)],$$
(3)

where Q(f) is the Fourier transform of signal q(t) and $Q^*(f)$ is the complex conjugate. Bicoherence is defined as the normalized bispectrum. In this work, the normalization established by Brillinger [27] was used. Following the notation of Refs. [28, 29], the bicoherence b is defined as

$$b^{2}(f_{1}, f_{2}) = \frac{|B(f_{1}, f_{2})|^{2}}{S(f_{1})S(f_{2})S(f_{1} + f_{2})}.$$
(4)

In this normalization, where S(f) is the PSD power, the magnitude of $b(f_1, f_2)$ is equivalent to a skewness function and not artifically bounded between 0 and 1 [28]. It is to be noted that the cross-bispectrum was computed for pairs



Z



Fig. 4 N factor contours

of two reconstructed pixel time signals, $q_x(t)$ and $q_{x+\Delta x}(t)$, spaced $\Delta x = 10$ pixels apart. The pairs were taken at discrete locations along the cone. As mentioned, the pixel signals were 23,000-35,000 points in length. Welch's method was employed, applying a Hanning window to segments 512 points in length with 50% overlap. Figs. 5-7 each show four realizations of cross-bicoherence *b* along with the associated average PSD calculated for each signal pair. In all cases, signals right above the wall were used for analysis. Shots 2989, 2988, and 2993 were chosen due to their higher frame rates. Similar to the N factor contours, artifacts of the frame rate and its harmonics would sometimes surface in the cross-bicoherence plots, manifesting as a grid of elliptical spots centered on (f_1, f_2) coordinates which were multiples of the frame rate f_{FR} , i.e. $(\{nf_{FR}\}_{n=1}^{\infty}, \{mf_{FR}\}_{m=1}^{\infty})$. These artifacts were removed from the results shown here. The axes on the cross-bicoherence maps are normalized by the corresponding second-mode peak frequency f_{2M} , which was identified at each location using the fit discussed earlier. The corresponding PSD curves show the same frequency domain but in kHz. All contours are plotted with the same intensity scale and enforce a threshold such that only frequency triads with $b \ge 0.3$ will appear. The symmetry lines $f_2 = f_1$ and $f_2 = -f_1$ are outlined on each map.

Cross-bicoherence for Shot 2989 is shown in Fig. 5. In this case, the 10-pixel Δx between cross-correlated signals corresponded to 1 mm. At the lowest $Re_{\infty} = 5.8 \times 10^6$, this shot exhibits the smallest number of prominent nonlinear interactions. At x = 720 mm, two interactions involving the second-mode fundamental f_{2M} and its first harmonic are visible in Fig. 5(a): one self-sum interaction $\langle f_{2M}, f_{2M}, 2f_{2M} \rangle$, and one difference interaction $\langle 2f_{2M}, -f_{2M}, f_{2M} \rangle$. The PSD curve also shows a small peak at the first harmonic, 2450 kHz. At x = 740 mm, the fundamental/first-harmonic interactions decrease in b, as does the first-harmonic peak (now at 2440 kHz) in the PSD. As shown in Fig. 5(b), triads involving the subharmonic $\frac{1}{2}f_{2M}$ emerge, namely the sum $\frac{1}{2}f_{2M}$, $\frac{1}{2}f_{2M}$, f_{2M} and difference $\frac{f_{2M}}{f_{2M}}$, $\frac{1}{2}f_{2M}$ interactions. As discussed by Shiplyuk in Ref. [30], this could be a sign of subharmonic resonance with the fundamental f_{2M} . By x = 754 mm, all interactions mentioned so far are present, and new fundamental/subharmonic interactions arise: the sum $\langle f_{2M}, \frac{1}{2}f_{2M}, \frac{3}{2}f_{2M} \rangle$ and difference $\langle \frac{3}{2}f_{2M}, -\frac{1}{2}f_{2M}, f_{2M} \rangle$. Additionally, Fig. 5(c) demonstrates phase locking between the fundamental and low frequencies $(f_{2M}, 0 \pm 0.05 f_{2M})$ as well as triads which are not locked to multiples of the fundamental or subharmonic. For example, the first-harmonic difference interaction appears more stretched around $f_1 = 2f_{2M}$, and discrete difference interactions in the region $(1.4 \le f_1/f_{2M} \le 2.1, -1 \le f_2/f_{2M} \le -0.5)$ can be identified. These triads provide context for the second-mode peak in the PSD which not only grows in power but also expands to include more lower-f content. Just 4 mm downstream, the subharmonic interactions have disappeared, as shown for x = 758 mm in Fig. 5(d). The fundamental/first-harmonic and the fundamental/low-frequency interactions increase in b, and now disturbances with frequency content at 90% f_{2M} exhibit phase locking: for example, sum and difference interactions appear for $(0.9f_{2M}, 0.9f_{2M}, 1.8f_{2M})$ and $(1.8f_{2M}, -0.9f_{2M}, 0.9f_{2M})$, respectively. These interactions likely represent the peak at 950 kHz protruding out of the second-mode band in the PSD curve.

Fig. 6 shows the cross-bicoherence computed at four locations along the cone for Shot 2988, at $Re_{\infty} = 6.35 \times 10^6$ m¹. In this case, Δx between cross-correlated signals was 1.5 mm. The most dominant interactions on the upstream end of the field of view are those in involving disturbances on the lower sideband of the fundamental in the range $0.7 \le f/f_{2M} \le 1$. This frequency band interacts with low-frequency content, as seen at $(0.75 \le f_1/f_{2M} \le 1, f_2/f_{2M} = 0 \pm 0.1)$ and $(0.7 \le f_1/f_{2M} \le 1, -1 \le f_2/f_{2M} \le -0.7)$ interactions. Self-sum interactions at $(0.85 \le f_1/f_{2M} \le 0.95, -0.7)$ $0.85 \le f_2/f_{2M} \le 0.95$) generate disturbances just below the first harmonic peak at $1.7f_{2M}$ and $1.9f_{2M}$ which, in turn, destructively interact with the $0.7 - 1.0 f_{2M}$ frequency band. These interactions likely represent the lobe of content in the range $800 \le f \le 1200$ kHz at $3.8 \le Re \le 3.95 \times 10^6$ in the ΔN contour of Fig. 4(b). It is interesting to note that lower-f content is seen in this region of the contour but mostly disappears for the range $3.95 \le Re \le 4.35 \times 10^6$. Moving downstream, the locus of phase locking in the off-fundamental band shrinks and the second-mode PSD peak grows, as can be seen in the PSD in Fig. 6(b). The highest level of b at x = 680 mm is within the difference interaction between the first harmonic and the second-mode fundamental, but the fundamental/low-frequency interaction still persists, limited to a narrower frequency band. It is to be noted that cross-bicoherence analysis performed using FLDI in Ref. [21] also identified strong fundamental/first-harmonic interactions at this streamwise location for a shot with the same Re_{∞} . At this point, the second-mode peak has reached spectral saturation, and disturbances moving downstream begin to break down to turbulence, as can be seen in Fig. 4(d) for $Re > 4.4 \times 10^6$. At x = 701 mm, the dominant interactions involve disturbances with $0.6 \le f/f_{2M} \le 0.8$. This band self-interacts to produce disturbances with frequency content $1.2 \le f/f_{2M} \le 1.5$, which undergoes sum and difference interactions with the $0.6 \le f/f_{2M} \le 0.8$ band. Simultaneously these two non-fundamental bands exhibit phase locking with the second-mode first harmonic, which can be seen in the PSD at 2500 kHz. Triads centered around $\langle 2f_{2M}, -0.7f_{2M}, 1.3f_{2M} \rangle$ and $\langle 2f_{2M}, -1.3f_{2M} \rangle$ $1.3 f_{2M}$ are also observed. At the final point, x = 720 mm, past the onset of breakdown, the second-mode peak has disappeared in the PSD of Fig. 6(d). At this point, there are many interactions with no obvious peaks, indicating the onset of turbulence.



Fig. 5 Cross-bicoherence for Shot 2989, $Re_{\infty} = 5.80 \times 10^6 \text{ m}^{-1}$, $R_N = 2 \text{ mm}$



Fig. 6 Bicoherence for Shot 2988, $Re_{\infty} = 6.35 \times 10^6 \text{ m}^{-1}$, $R_N = 2 \text{ mm}$

Fig. 7 displays the cross-bicoherence maps and PSD curves at four locations along the cone for Shot 2993 with the bluntest nosetip, $R_N = 3$ mm. Signal pairs from this shot were separated by $\Delta x = 1.6$ mm. On the upstream end at x = 621 mm, faint nonlinear interactions are visible. As shown in Fig. 7(a), frequencies near the second-mode peak f_{2M} exhibit a self-sum interaction $(0.9f_{2M}, 0.9f_{2M}, 1.8f_{2M})$ as well as phase locking with low-frequency content $(0.95f_{2M}, 1.8f_{2M})$ $0 \pm 0.05 f_{2M}$, $1 \pm 0.1 f_{2M}$). In the PSD, a peak at 950 kHz is just as strong as the second-mode peak, and this content is likely interacting in the $(0.8f_{2M}, -0.4f_{2M}, 0.4f_{2M})$ triad seen on the cross-bicoherence map. By x = 670 mm, the interactions grow in strength. The 1190-kHz f_{2M} peak now exceeds that at 900 kHz in the PSD, and fundamental/low-f interactions are visible at $(1 \pm 0.05 f_{2M}, 0 \pm 0.07 f_{2M})$ and $(1 \pm 0.05 f_{2M}, -1 \pm 0.05 f_{2M})$. Disturbances with content $0.8 \le f/f_{2M} \le 0.9$ demonstrate a higher level of phase locking with low-f content, however, generating triads located at $(0.8 \le f_1/f_{2M} \le 0.9, f_2/f_{2M} = 0 \pm 0.1)$ and $(0.8 \le f_1/f_{2M} \le 0.9, -0.9 \le f_2/f_{2M} \le -0.8)$. At x = 705 mm, second-mode content has saturated and disturbances approach breakdown, which can be associated with the spectral spreading near $Re = 5 \times 10^6$ in the ΔN contour of Fig. 4(e). Peaks at $f_{2M} = 1140$ kHz and 900 kHz are still visible in the PSD of Fig. 7(c), but a subharmonic $(1/2f_{2M})$ peak rises near 540 kHz. In the cross-bicoherence map, phase-locked interactions at $(0.8 \le f_1/f_{2M} \le 0.9, f_2/f_{2M} = 0 \pm 0.1)$ persist, but interactions involving subharmonic content exhibit the highest b, i.e. the triads located at $(0.4 \le f_1/f_{2M} \le 0.6, f_2/f_{2M} = 0 \pm 0.1)$ and $(0.4 \le f_1/f_{2M} \le 0.6, f_2/f_{2M} \le 0.6)$ $-0.6 \le f_2/f_{2M} \le -0.4$). The subharmonic also interacts with the fundamental through sum $\langle 1/2f_{2M}, 1/2f_{2M}, f_{2M} \rangle$ and difference $\langle f_{2M}, -\frac{1}{2}f_{2M}, \frac{1}{2}f_{2M} \rangle$ triads. Finally, at x = 720 mm, past the onset of breakdown, the PSD shows a broadband elevation of power and cross-bicoherence is widely distributed over the frequency domain.



Fig. 7 **Bicoherence for Shot 2993,** $Re_{\infty} = 7.04 \times 10^6 \text{ m}^{-1}$, $R_N = 3 \text{ mm}$

Summarizing these observations, the cross-bicoherence maps demonstrate distinct sequences of nonlinear interaction for the three runs. For the sharper-nose, lowest- Re_{∞} case, where no breakdown is observed, nonlinear development is tightly confined to fundamental, first-harmonic, and subharmonic interactions. Fundamental/first-harmonic interactions begin, fundamental/subharmonic interactions follow, and finally fundamental/low-f interactions emerge. In the sharper-nose, higher- Re_{∞} case, a thick band of near-fundamental ($0.7 - 1.0f_{2M}$) interactions precede, generating a lobe of low-f content in the ΔN contour. Moving downstream, the extent of nonlinear interaction shrinks, as does the low-f lobe in the ΔN contour, but fundamental/first-harmonic and fundamental/low-f interactions persist. At the point of breakdown, dominant interactions involve frequency content in the range $0.6 - 0.8f_{2M}$, which interact with $1.2 - 1.5f_{2M}$ as well as the second-mode first harmonic. In the blunter-nose case, low-f interactions are the dominant nonlinear mechanism. First, the second-mode fundamental interacts with low-f content, and then a frequency band $0.8 - 0.9 f_{2M}$ interacts with low-f content, and finally, near the point of breakdown, interactions between second-mode subharmonic and low-frequency content emerge.

IV. Conclusion

Schlieren imaging was used to characterize the transitional boundary layer on a 5° cone in hypervelocity flow. Tests at various freestream Reynolds numbers and two different nose bluntnesses were conducted. Distinct structures were identified: second-mode wavepackets, turbulent spots extending into the freestream, and nonmodal features propagating in the bluntest case. PSD estimates were generated using a time-reconstruction technique, and peak frequencies between 1200-1300 kHz were associated with second-mode content within the boundary layer. For more developed boundary layers, this peak extended higher from the wall. For the bluntest-nose case, the second-mode peak was only visible right above the wall, but other spectral peaks emerged higher in the boundary layer. N-factor contours demonstrated the broadening of the second-mode band with Reynolds number for the sharper-nose cases and motivated an investigation into the interplay between second-mode content and outer-band content. Cross-bicoherence calculations indicated that the sharper-nose cases experience nonlinear interactions between the second-mode fundamental and its first harmonic until the point of breakdown. For a sharp-nose laminar run, fundamental/subharmonic interactions also developed but diminished as fundamental/low-frequency interactions emerged. For the sharp-nose transitional run, fundamental/first-harmonic and fundamental/low-frequency interactions existed up to the point of breakdown, where interactions involving content at 60-80% of the second-mode frequency showed the highest levels of cross-bicoherence. For the blunt-nose transitional case, low-frequency interactions constituted the primary nonlinear mechanism. The second-mode fundamental interacted with low-frequency content, before interactions between disturbances at 80-90% of the second-mode frequency interacted and low-frequency content become prominent. At the point of breakdown, the second-mode subharmonic interacted with the low-frequency band.

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